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BUBBLE DEFORMATION IN AN ELECTRIC FIELD

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The problem of expansion of bubbles in a compressible liquid upon application of an electric field is solved. The effect of the electric field on phase transition is evaluated.

It has been experimentally established [1, 2] that under the influence of an electric field bubbles and droplets suspended in a dielectric field expand in the direction along the field independent of the ratio between dielectric permittivities of the medium ε_2 and the inclusions ε_1. A theoretical description of this effect is of interest in refining the peculiarities of heat exchange in an electric field [3].

The existing calculations of bubble deformation [1, 2] do not agree with experimental results without the assumption that the electrostriction pressure in the liquid is "fictitious" [1] or that the coefficient of liquid surface tension is strongly dependent on the electric field vector [2].

We will demonstrate that consideration of the compressibility of the liquid in solution of the equation of equilibrium of the interphase boundary leads to results which agree qualitatively with experimental data

$$\frac{1}{R_1}+\frac{1}{R_2}=\frac{P_1(\rho_1)-P_2(\rho_2)}{\delta}-\frac{\varepsilon_0(\varepsilon_2-\varepsilon_1)}{2\delta}\left(\frac{\varepsilon_1}{\varepsilon_2}E_{n1}^2+E_{t1}^2\right)+\rho_2\frac{\partial\varepsilon_2}{\partial\rho_2}\frac{\varepsilon_0E_2^2}{2\delta}-\rho_1\frac{\partial\varepsilon_1}{\partial\rho_1}\frac{\varepsilon_0E_1^2}{2\delta}, \tag{1}$$

where ρ_1, ρ_2 are densities; P_1, P_2, pressures; E_{t1}, E_{n1}, tangential and normal components of the field within the inclusion on the boundary with the medium; E_1, E_2, field intensities; δ, surface tension; R_1, R_2, major radii of curvature of the surface.

In contrast to the case of an incompressible liquid Eq. (1) is insufficient for determination of the bubble deformation, since the liquid density becomes variable in an inhomogeneous electric field as produced by the bubble. Electrostriction causes flow of some of the liquid from a region of weak field to a region of strong field, with the density and hydrostatic pressure increasing at the equator and decreasing at the poles of the bubble in comparison to the analogous parameters far from the bubble. After establishment of a stationary state the pressure distribution is described by the equation [4]

$$P_2(\rho_2)-\rho_2\frac{\partial\varepsilon_2}{\partial\rho_2}\frac{\varepsilon_0E_2^2}{2}=P_0, \tag{2}$$

where P_0 is a quantity dependent on the geometry of the electrode system creating the field, and the conditions at the boundary of the liquid volume. In experiments, apparently, case (a) is often realized, wherein the field occupies a small portion of the liquid volume and liquid flow is not inhibited at all. Then P_0 is equal to the hydrostatic pressure P_{ex}, existing in the liquid before field application. In case (b), where the influx of liquid is impossible or has not occurred

$$P_0=P_{ex}-\rho_2\frac{\partial\varepsilon_2}{\partial\rho_2}\frac{\varepsilon_0E_0^2}{2}. \tag{3}$$

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After substitution of Eq. (2) in Eq. (1) and simple transformations, it can be proved that the equation obtained describes an ellipsoid of revolution of low eccentricity.

We define the ellipsoid parameters: the relative elongation along the field $\Delta_1 = (a - R_0)/R_0$ and the relative compression at the equator $\Delta_2 = (R_0 - b)/R_0$, where a and b are the ellipsoid semiaxes and R_0 is the bubble radius in the absence of the field. The gas within the bubble is considered ideal and electrostriction in the gas is neglected

$$\Delta_1 = \left[\frac{P_{ex}R_0\varepsilon_0(\varepsilon_2-\varepsilon_1)^2E_1^2}{4\varepsilon_2\delta} - \frac{\varepsilon_0(\varepsilon_2-\varepsilon_1)\varepsilon_1E_1^2}{2\varepsilon_2} + P_{ex} - P_0\right]\left(3P_{ex} + \frac{4\delta}{R_0}\right)^{-1}, \quad (4)$$

$$\Delta_2 = \left[\frac{P_{ex}R_0\varepsilon_0(\varepsilon_2-\varepsilon_1)^2E_1^2}{8\varepsilon_2\delta} + \frac{\varepsilon_0(\varepsilon_2-\varepsilon_1)E_1^2}{2} - P_{ex} + P_0\right]\left(3P_{ex} + \frac{4\delta}{R_0}\right)^{-1}.$$

It follows from Eq. (4) that the ellipslid eccentricity $\Delta_1 + \Delta_2$ is always positive, i.e., bubbles in an electric field have the form of prolate ellipsoids of revolution, the major axis of which is oriented along the field.

We will consider variants of bubble behavior. In case (a) small bubbles ($R_0 < 2\delta\varepsilon_1 \cdot [P_{ex}(\varepsilon_2 - \varepsilon_1)]^{-1}$) are compressed both across the equator and in the longitudinal direction, while large bubbles, contracting across the equator, extend themselves in the direction of the field. It is significant that the bubble volume then decreases: $\Delta V/V = \Delta_1 - 2\Delta_2 < 0$. This consequence proves quite important in the consideration of liquid–vapor equilibrium. With increase in density of the internal phase, developing because of bubble compression, the chemical potential of the vapor increases. Decrease in the chemical potentials of the phases in the external field may be neglected [4], so that the difference between the chemical potentials of vapor and liquid also increases, reducing the thermodynamic stability of the vapor and increasing the stability of the liquid phase. The phase equilibrium curve must shift in the direction of higher temperatures, which is indirectly supported by experimental data on increase in thermal fluxes upon boiling in an electric field [5].

In case (b) the bubble extends in the longitudinal direction, and apparently shrinks in the transverse direction. The degree of accuracy presently attained in calculation of $\partial\varepsilon/\partial\rho$ is insufficient for an unambiguous answer to this question. As for the bubble volume, it should increase, so that the chemical potential decreases and the phase equilibrium curve shifts in the direction of lower temperature.

NOTATION

ρ_1, ρ_2, densities of the internal and external phases, respectively; P_1, P_2, pressures; ε_1, ε_2, dielectric permittivities; E_1, E_2, field intensities; E_{t1}, E_{n1}, tangential and normal field components within the inclusion at the phase boundary; δ, surface tension coefficient; R_1, R_2, major radii of curvature of the surface; P_0, pressure in the liquid in the weak field region; P_{ex}, external pressure; R_0, initial bubble radius; Δ_1, Δ_2, relative elongation along field and relative compression across equator; $\Delta V/V$, relative change in bubble volume; a, b, ellipsoid semiaxes.

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